Roll No. .... Total Pages : 3

BT-I/D-19 **31001**

MATHEMATICS–I

Paper : Math-101E

Opt. : II

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions in all selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Solve using Maclaurin's series :

$$\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \ldots\ldots$$ 10

(b) Find the asymptotes of the curve

$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0.$ 10

2. (a) Show that the radius of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $\frac{a}{\sqrt{2}}$. 10

(b) Trace the curve $ay^2 = x^2(a - x)$. 10

UNIT–II

3. (a) If $v = \log(x^2 + y^2 + z^2)$, prove that

$$(x^2 + y^2 + z^2)\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] = 2.$$ 10

(b) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$, prove that

$$x^2 . \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\sin^2 u - \sin 2u.$$ 10

4. (a) If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 10

(b) By method of differentiation under integral sign, prove that $\int_0^{\pi} \frac{\log(1 + \alpha\cos x)}{\cos x}\, dx = \pi \sin^{-1}\alpha.$ 10

UNIT–III

5. (a) Solve $\int_0^{\infty}\int_0^{x} xe^{-x^2/y}\, dy\, dx$. 10

(b) Evaluate $\int_0^1 \int_{e^x}^{e} \frac{dy\,dx}{\log y}$ by changing order of integration. 10

6. (a) Evaluate the integral $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy\,dx\,dz$. 10

(b) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. 10

UNIT–IV

7. (a) Find the directional derivative of $\phi = xy^2z + 4xz^2$ at the point (1, –2, –1) in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$. 10

(b) Give the physical interpretation of divergence. 10

8. (a) If $F=(5xy-6x^2)\hat{i}+(2y-4x)\hat{j}$, evaluate $\int_C F.dh$ along the curve C in the xy-plane $y = x^3$ from the point (1, 1) to (2, 8). 10

(b) Using Stoke's theorem, evaluate

$$\oint_C (yz\,dx + zx\,dy + xy\,dz),$$

where C is the curve $x^2 + y^2 = 1$, $z = y^2$. 10